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Preface

Kindergarten to Grade 8 Mathematics Curriculum Framework: 2013 Revisions is a document that reflects revisions that have been made to the Number strand of the 2008 Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes. Manitoba Education has worked with provincial stakeholders to reinforce the importance of conceptual understanding, procedural thinking, and problem solving, and to clarify grade-level expectations. Revisions were made to the introduction and to some of the learning outcomes and achievement indicators in the Number strand.
The Western Canadian Protocol for Collaboration in Basic Education Kindergarten to Grade 12 was signed December 1993 by the Ministers of Education from Alberta, British Columbia, Manitoba, Northwest Territories, Saskatchewan, and Yukon Territory. In February 2000, following the addition of Nunavut, the protocol was renamed the Western and Northern Canadian Protocol (WNCP) for Basic Education.

In 2005, the Ministers of Education from all the WNCP jurisdictions unanimously concurred with the rationale of the original partnership because of the importance placed on

- common educational goals
- the ability to collaborate to achieve common goals
- high standards in education
- planning an array of educational opportunities
- removing obstacles to accessibility for individual learners
- optimum use of limited educational resources

The Common Curriculum Framework for K–9 Mathematics was developed by the seven ministries of education in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others.

The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. Each of the provinces and territories will determine when and how the framework will be implemented within its own jurisdiction.
**Introduction**

**Purpose of the Document**

This document provides a common base for the curriculum expectations mandated by Manitoba Education, Citizenship and Youth, which will result in consistent student outcomes in mathematics across Manitoba and enable easier transfer for students moving from one region to another. Its intent is to clearly communicate high expectations for students in mathematics education to all education partners across Manitoba, and to facilitate the development of common learning resources.

**Beliefs about Students and Mathematics Learning**

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences. Students learn by attaching meaning to what they do, and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics. Students need frequent opportunities to develop and reinforce their conceptual understanding, procedural thinking, and problem solving abilities. By addressing these three interrelated components, students will strengthen their ability to apply mathematical learning to their daily lives.

The learning environment should value and respect all students’ experiences and ways of thinking, so that...
learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

**First Nations, Métis, and Inuit Perspectives**

First Nations, Métis, and Inuit students in Manitoba come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings including urban, rural, and isolated communities. Teachers need to understand the diversity of cultures and experiences of students.

First Nations, Métis, and Inuit students often have a whole-world view of the environment in which they live, and learn best in a holistic way. This means that students look for connections in learning, and learn best when mathematics is contextualized and not taught as discrete components.

First Nations, Métis, and Inuit students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding.

It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students.

The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks).

**Affective Domain**

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

Teachers, students, and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective
domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

**Early Childhood**

Young children are naturally curious and develop a variety of mathematical ideas before they enter kindergarten. Children make sense of their environment through observations and interactions at home, in daycares, preschools, and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home.

Activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are actively engaged in their environment.

Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

**Goals for Students**

The main goals of mathematics education are to prepare students to

- communicate and reason mathematically
- **use mathematics confidently, accurately, and efficiently to solve problems**
- appreciate and value mathematics
- **make connections between mathematical knowledge and skills and their applications**
- commit themselves to lifelong learning
- **become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world**

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity
The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

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Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip-counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 184)

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.
Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (BC Ministry of Education 146).

Number sense is an awareness and understanding of what numbers are, their relationships, their magnitude, and the relative effect of operating on numbers, including the use of mental mathematics and estimation (Fennell and Landis 187).

Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. Students who have number sense are computationally fluent, are flexible with numbers, and have intuition about numbers. Number sense evolves and typically results as a by-product of learning rather than through direct instruction. Number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics.

These skills contribute to students’ interaction with and understanding of their environment.

Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps develop students’ algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The discovery of possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.
Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.
Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The Common Curriculum Framework incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, and written and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated.
Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. “Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding. . . Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine 5).

**Mental Mathematics and Estimation [ME]**

Mental mathematics and estimation is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids. It improves computational fluency by developing efficiency, accuracy, and flexibility.

Mental mathematics and estimation are fundamental concepts of number sense.

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein 442).

Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope V).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgments and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to know which strategy to use and how to use it.

To help students become efficient with computational fluency, students need to develop mental math skills and recall math facts automatically. Learning math facts is a developmental process where the focus of instruction is on thinking and building number relationships. Facts become automatic for students through repeated exposure and practice. When a student recalls facts, the answer should be produced without resorting to inefficient means, such as counting. When facts are automatic, students are no longer using strategies to retrieve them from memory.

**Problem Solving [PS]**

“Problem solving is an integral part of all mathematics learning” (NCTM, Problem Solving). Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you . . . ?” or “How could you . . . ?”, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.
In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior knowledge in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives, and develops confident, cognitive, mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes, and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Technology has the potential to enhance the teaching and learning of mathematics. It can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. Students need to know when it is appropriate to use technology such as a calculator and when to apply their mental computation, reasoning, and estimation skills to predict and check answers. The use of technology can enhance, although it should not replace, conceptual understanding, procedural thinking, and problem solving throughout Kindergarten to Grade 8. While technology can be used in Kindergarten to Grade 3 to enrich learning, it is expected that students will meet all outcomes without the use of calculators.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world” (Armstrong 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and to know several estimation strategies (Shaw and Cliatt 150).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.
Strands

The learning outcomes in the Manitoba Curriculum Framework are organized into four strands across the grades, K–9. Some strands are further subdivided into substrands. There is one general learning outcome per substrand across the grades, K–9.

The strands and substrands, including the general learning outcome for each, follow.

Number
- Develop number sense.

Patterns and Relations
Patterns
- Use patterns to describe the world and solve problems.

Variables and Equations
- Represent algebraic expressions in multiple ways.

Shape and Space

Measurement
- Use direct and indirect measure to solve problems.

3-D Objects and 2-D Shapes
- Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Transformations
- Describe and analyze position and motion of objects and shapes.

Statistics and Probability

Data Analysis
- Collect, display, and analyze data to solve problems.

Chance and Uncertainty
- Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.
Learning Outcomes and Achievement Indicators

The *Manitoba Curriculum Framework* is stated in terms of general **learning outcomes**, specific **learning outcomes**, and achievement indicators.

**General learning outcomes** are overarching statements about what students are expected to learn in each strand/substrand. The general **learning outcome** for each strand/substrand is the same throughout the grades.

**Specific learning outcomes** are statements that identify the specific skills, understanding, and knowledge students are required to attain by the end of a given grade.

**Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success.

In this document, the word “including” indicates that any ensuing items **must be addressed** to fully meet the learning outcome. The phrase “such as” indicates that the ensuing items are provided for illustrative purposes or clarification, and are **not requirements that must be addressed** to fully meet the learning outcome.

Summary

The conceptual framework for K–8 mathematics describes the nature of mathematics, mathematical processes, and the mathematical concepts to be addressed in Kindergarten to Grade 8 mathematics. The components are not meant to stand alone. Learning activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes, and lead students to an understanding of the nature of mathematics through specific knowledge, skills, and attitudes among and between strands.
Instructional Focus

The *Manitoba Curriculum Framework* is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of learning outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Routinely incorporating conceptual understanding, procedural thinking, and problem solving within instructional design will enable students to master the mathematical skills and concepts of the curriculum.
- Integration of the mathematical processes within each strand is expected.
- Problem solving, conceptual understanding, reasoning, making connections, and procedural thinking are vital to increasing mathematical fluency, and must be integrated throughout the program.
- Concepts should be introduced using manipulatives and gradually developed from the concrete to the pictorial to the symbolic.
- Students in Manitoba bring a diversity of learning styles and cultural backgrounds to the classroom and they may be at varying developmental stages. Methods of instruction should be based on the learning styles and abilities of the students.
- Use educational resources by adapting to the context, experiences, and interests of students.
- Collaborate with teachers at other grade levels to ensure the continuity of learning of all students.
- Familiarize yourself with exemplary practices supported by pedagogical research in continuous professional learning.
- Provide students with several opportunities to communicate mathematical concepts and to discuss them in their own words.

“Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students” (Ontario 24).
Please view this document in colour.

The learning outcomes that have been revised are stated with the current learning outcome on the top of the chart followed by the proposed change directly underneath highlighted in yellow.

Brown type indicates an achievement indicator that has moved from one grade to another. Moved achievement indicators are now learning outcomes.
### Addition and Subtraction Facts to 18

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Current</strong> 1.N.10. Describe and use mental mathematics strategies (memorization not intended), including</td>
<td>(It is not intended that students recall the basic facts but become familiar with strategies to mentally determine sums and differences. Students should show their strategy using manipulatives or visual aids.)</td>
</tr>
<tr>
<td>■ counting on or counting back</td>
<td>■ Use and describe a personal strategy for determining a sum.</td>
</tr>
<tr>
<td>■ using one more or one less</td>
<td>■ Use and describe a personal strategy for determining a difference.</td>
</tr>
<tr>
<td>■ making 10</td>
<td>■ Describe and write the related subtraction fact for an addition fact.</td>
</tr>
<tr>
<td>■ starting from known doubles</td>
<td>■ Describe and write the related addition fact for a subtraction fact.</td>
</tr>
<tr>
<td>■ using addition to subtract to determine the basic addition and related subtraction facts to 18.</td>
<td></td>
</tr>
<tr>
<td>[C, CN, ME, PS, R, V]</td>
<td></td>
</tr>
<tr>
<td><strong>Revisions</strong> 1.N.10. Describe and use mental mathematics strategies including</td>
<td>(It is intended that students show their understanding of strategies using manipulatives, pictorial representations, and/or patterns when determining sums and differences.)</td>
</tr>
<tr>
<td>■ counting on, counting back</td>
<td>■ Use and describe a mental mathematics strategy for determining a sum.</td>
</tr>
<tr>
<td>■ using one more, one less</td>
<td>■ Use and describe a mental mathematics strategy for determining a difference.</td>
</tr>
<tr>
<td>■ making 10</td>
<td>■ Use and describe the related addition facts for a subtraction fact (fact family) (e.g., 6 – 4 = 2 has two related addition facts: 4 + 2 = 6, 2 + 4 = 6).</td>
</tr>
<tr>
<td>■ starting from known doubles</td>
<td>■ Use and describe related subtraction facts for an addition fact (fact family) (e.g., 2+3=5 has two related subtraction facts: 5 - 3 = 2, 5 - 2 = 3).</td>
</tr>
<tr>
<td>■ using addition to subtract to determine the basic addition and related subtraction facts to 18.</td>
<td></td>
</tr>
<tr>
<td>[C, CN, ME, PS, R, V]</td>
<td></td>
</tr>
</tbody>
</table>
### Addition and Subtraction Facts to 18 (continued)

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current 2.N.10. Apply mental mathematics strategies, including</td>
<td>Explain the mental mathematics strategy that could be used to determine an addition or subtraction fact, such as</td>
</tr>
<tr>
<td>■ using doubles</td>
<td>■ using doubles (e.g., for 4 + 6, think 5 + 5)</td>
</tr>
<tr>
<td>■ making 10</td>
<td>■ using doubles plus one (e.g., for 4 + 5, think 4 + 4 + 1)</td>
</tr>
<tr>
<td>■ using one more, one less</td>
<td>■ using doubles take away one (e.g., for 4 + 5, think 5 + 5 – 1)</td>
</tr>
<tr>
<td>■ using two more, two less</td>
<td>■ using doubles plus two (e.g., for 4 + 6, think 4 + 4 + 2)</td>
</tr>
<tr>
<td>■ building on a known double</td>
<td>■ using doubles take away two (e.g., for 4 + 6, think 6 + 6 – 2)</td>
</tr>
<tr>
<td>■ using addition for subtraction</td>
<td>■ making 10 (e.g., for 7 + 5, think 7 + 3 + 2)</td>
</tr>
<tr>
<td>to develop recall of basic addition facts to 18 and related subtraction facts.</td>
<td>■ building on a known double (e.g., 6 + 6 = 12, so 6 + 7 = 12 + 1 = 13)</td>
</tr>
<tr>
<td>[C, CN, ME, R, V]</td>
<td>■ using addition for subtraction (e.g., for 7 – 3, think 3 + ? = 7)</td>
</tr>
<tr>
<td></td>
<td>■ Use and describe a personal strategy for determining a sum to 18 and the corresponding subtraction.</td>
</tr>
</tbody>
</table>

**Revisions**

| 2.N.10. Apply mental mathematics strategies, including | Explain the mental mathematics strategy that could be used to determine an addition or subtraction fact, such as |
| ■ using doubles | ■ using doubles (e.g., for 4 + 6, think 5 + 5) |
| ■ making 10 | ■ using doubles plus one (e.g., for 4 + 5, think 4 + 4 + 1) |
| ■ using one more, one less | ■ using doubles take away one (e.g., for 4 + 5, think 5 + 5 – 1) |
| ■ using two more, two less | ■ using doubles plus two (e.g., for 4 + 6, think 4 + 4 + 2) |
| ■ building on a known double | ■ using doubles take away two (e.g., for 4 + 6, think 6 + 6 – 2) |
| ■ using addition for subtraction | ■ making 10 (e.g., for 7 + 5, think 7 + 3 + 2) |
| to develop recall of basic addition facts to 18 and related subtraction facts. | ■ building on a known double (e.g., 6 + 6 = 12, so 6 + 7 = 12 + 1 = 13) |
| [C, CN, ME, R, V] | ■ using addition for subtraction (e.g., for 7 – 3, think 3 + ? = 7) |
| **Recall of facts to 10, doubles to 9 + 9, and related subtraction facts is expected by the end of Grade 2.** | ■ Use and describe a personal strategy for determining a sum to 18 and the corresponding subtraction. |
Addition and Subtraction Facts to 18 *(continued)*

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
</tr>
</thead>
</table>
| Current                     | ■ Describe a mental mathematics strategy that could be used to determine a given basic fact, such as  
■ doubles (e.g., for 6 + 8, think 7 + 7)  
■ doubles plus one (e.g., for 6 + 7, think 6 + 6 + 1)  
■ doubles take away one (e.g., for 6 + 7, think 7 + 7 – 1)  
■ doubles plus two (e.g., for 6 + 8, think 6 + 6 + 2)  
■ doubles take away two (e.g., for 6 + 8, think 8 + 8 – 2)  
■ making 10 (e.g., for 6 + 8, think 6 + 4 + 4 or 8 + 2 + 4)  
■ commutative property (e.g., for 3 + 9, think 9 + 3)  
■ addition to subtraction (e.g., for 13 – 7, think 7 + ? = 13)  
■ Provide a rule for determining answers for adding and subtracting zero.  
■ Recall doubles to 18 and related subtraction facts.  
■ Recall compatible number pairs for 5 and 10.  
| Recall of addition and related subtraction facts to 18. is expected by the end of Grade 3. |  
| Revisions                    | ■ Describe a mental mathematics strategy that could be used to determine a given basic fact, such as  
■ doubles (e.g., for 6 + 8, think 7 + 7)  
■ doubles plus one (e.g., for 6 + 7, think 6 + 6 + 1)  
■ doubles take away one (e.g., for 6 + 7, think 7 + 7 – 1)  
■ doubles plus two (e.g., for 6 + 8, think 6 + 6 + 2)  
■ doubles take away two (e.g., for 6 + 8, think 8 + 8 – 2)  
■ making 10 (e.g., for 6 + 8, think 6 + 4 + 4 or 8 + 2 + 4)  
■ commutative property (e.g., for 3 + 9, think 9 + 3)  
■ addition to subtract (e.g., for 13 – 7, think 7 + ? = 13)  
■ Provide a rule for determining answers for adding and subtracting zero.  

These have been moved to 1.N.10 and 2.N.10.
### Multiplication and Division Facts to 81

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Current</strong> 3.N.11. Demonstrate an understanding of multiplication to 5 × 5 by</td>
<td>(It is not intended that students recall the basic facts but become familiar with strategies to mentally determine products.)</td>
</tr>
<tr>
<td>■ representing and explaining multiplication using equal grouping and arrays</td>
<td>■ Identify events from experience that can be described as multiplication.</td>
</tr>
<tr>
<td>■ creating and solving problems in context that involve multiplication</td>
<td>■ Represent a story problem (orally, shared reading, written) using manipulatives or diagrams, and record in a number sentence.</td>
</tr>
<tr>
<td>■ modelling multiplication using concrete and visual representations, and recording the</td>
<td>■ Skip-count by 2s, 3s, 4s, and 5s to determine the answer to a multiplication problem represented as equal groups.</td>
</tr>
<tr>
<td>process symbolically</td>
<td>■ Represent a multiplication expression as repeated addition.</td>
</tr>
<tr>
<td>■ relating multiplication to repeated addition</td>
<td>■ Represent a repeated addition as multiplication.</td>
</tr>
<tr>
<td>■ relating multiplication to division</td>
<td>■ Create and illustrate a story problem for a number sentence.</td>
</tr>
<tr>
<td>[C, CN, PS, R]</td>
<td>■ Represent, concretely or pictorially, equal groups for a number sentence.</td>
</tr>
<tr>
<td></td>
<td>■ Represent a multiplication expression using an array.</td>
</tr>
<tr>
<td></td>
<td>■ Create an array to model the commutative property of multiplication.</td>
</tr>
<tr>
<td></td>
<td>■ Relate multiplication to division by using arrays and by writing related number sentences.</td>
</tr>
<tr>
<td></td>
<td>■ Solve a problem in context involving multiplication.</td>
</tr>
</tbody>
</table>

| **Revisions** 3.N.11. Demonstrate an understanding of multiplication to 5 × 5 by            | (It is intended that students show their understanding of strategies using manipulatives, pictorial representations, and/or patterns when determining products.) |
| ■ representing and explaining multiplication using equal grouping and arrays                | ■ Identify events from experience that can be described as multiplication.              |
| ■ creating and solving problems in context that involve multiplication                      | ■ Represent a story problem (orally, shared reading, written) using manipulatives or diagrams, and record in a number sentence. |
| ■ modelling multiplication using concrete and visual representations, and recording the    | ■ Skip-count by 2s, 3s, 4s, and 5s to determine the answer to a multiplication problem represented as equal groups. |
|    process symbolically                                                                    | ■ Represent a multiplication expression as repeated addition.                           |
| ■ relating multiplication to repeated addition                                              | ■ Represent a repeated addition as multiplication.                                     |
| ■ relating multiplication to division                                                       | ■ Create and illustrate a story problem for a number sentence.                          |
| [C, CN, PS, R]                                                                              | ■ Represent, concretely or pictorially, equal groups for a number sentence.             |
|                                                                                             | ■ Represent a multiplication expression using an array.                                 |
|                                                                                             | ■ Create an array to model the commutative property of multiplication.                  |
|                                                                                             | ■ Relate multiplication to division by using arrays and by writing related number sentences. |
|                                                                                             | ■ Solve a problem in context involving multiplication.                                  |
## Specific Learning Outcome(s)

<table>
<thead>
<tr>
<th>Current</th>
<th>3.N.12. Demonstrate an understanding of division by</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>■ representing and explaining division using equal sharing and equal grouping</td>
</tr>
<tr>
<td></td>
<td>■ creating and solving problems in context that involve equal sharing and equal grouping</td>
</tr>
<tr>
<td></td>
<td>■ modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically</td>
</tr>
<tr>
<td></td>
<td>■ relating division to repeated subtraction</td>
</tr>
<tr>
<td></td>
<td>■ relating division to multiplication (limited to division related to multiplication facts up to $5 \times 5$).</td>
</tr>
<tr>
<td></td>
<td>[C, CN, PS, R]</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Revisions</th>
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</tr>
<tr>
<td></td>
<td>[C, CN, PS, R]</td>
</tr>
</tbody>
</table>

## Achievement Indicators

- Identify events from experience that can be described as equal sharing.
- Identify events from experience that can be described as equal grouping.
- Illustrate, with counters or a diagram, a story problem involving equal sharing, presented orally or through shared reading, and solve the problem.
- Illustrate, with counters or a diagram, a story problem involving equal grouping, presented orally or through shared reading, and solve the problem.
- Listen to a story problem, represent the numbers using manipulatives or a sketch, and record the problem with a number sentence.
- Create, and illustrate with counters, a story problem for a number sentence.
- Represent a division expression as repeated subtraction.
- Represent a repeated subtraction as a division expression.
- Relate division to multiplication by using arrays and by writing related number sentences.
- Solve a problem involving division.

(Note: It is intended that students show their understanding of strategies using manipulatives, pictorial representations, and/or patterns when determining quotients.)
### Specific Learning Outcome(s)

**Current**

4.N.5. Describe and apply mental mathematics strategies, such as
- skip-counting from a known fact
- using doubling or halving
- using doubling and adding one more group
- using patterns in the 9s facts
- using repeated doubling
to develop recall of basic multiplication facts to 9 × 9 and related division facts.

[C, CN, ME, PS, R]

**Revisions**

4.N.5. Describe and apply mental mathematics strategies, such as
- skip-counting from a known fact
- using doubling, halving
- using doubling and adding one more group
- using patterns in the 9s facts
- using repeated doubling
to develop an understanding of basic multiplication facts to 9 × 9 and related division facts.

[C, CN, ME, PS, R]

Recall of the multiplication and related division facts up to 5 × 5 is expected by the end of Grade 4.

### Achievement Indicators

**Current**

- Provide examples for applying mental mathematics strategies:
  - doubling (e.g., for 4 × 3, think 2 × 3 = 6, and 4 × 3 = 6 + 6)
  - doubling and adding one more group (e.g., for 3 × 7, think 2 × 7 = 14, and 14 + 7 = 21)
  - use ten facts when multiplying by 9 (e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63)
  - halving (e.g., for 30 ÷ 6, think 15 ÷ 3 = 5)
  - relating division to multiplication (e.g., for 64 ÷ 8, think 8 × □ = 64)

**Revisions**

- Provide examples for applying mental mathematics strategies:
  - skip counting from a known fact (e.g., for 6 × 3, think 5 × 3 = 15, then 15 + 3 = 18)
  - doubling/halving (e.g., for 4 × 3, think 2 × 3 = 6, and 4 × 3 = 6 + 6)
  - using a known double and adding one more group (e.g., for 3 × 7, think 2 × 7 = 14, then 14 + 7 = 21)
  - repeated doubling (e.g., for 4 × 6, think 2 × 6 = 12 then 2 × 12 = 24)
  - use ten facts when multiplying by 9 (e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63)
  - halving (e.g., for 30 ÷ 6, think 15 ÷ 3 = 5)
  - relating division to multiplication (e.g., for 64 ÷ 8, think 8 × □ = 64)
## Multiplication and Division Facts to 81 (continued)

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
</tr>
</thead>
</table>
| **Current** 5.N.3. Determine multiplication facts (to 81) and related division facts. [C, CN, ME, R, V] | - Describe the mental mathematics strategy used to determine a basic fact, such as  
  - skip-count up by one or two groups from a known fact (e.g., if $5 \times 7 = 35$, then $6 \times 7$ is equal to $35 + 7$ and $7 \times 7$ is equal to $35 + 7 + 7$)  
  - skip-count down by one or two groups from a known fact (e.g., if $8 \times 8 = 64$, then $7 \times 8$ is equal to $64 - 8$ and $6 \times 8$ is equal to $64 - 8 - 8$)  
  - doubling (e.g., for $8 \times 3$ think $4 \times 3 = 12$, and $8 \times 3 = 12 + 12$)  
  - patterns when multiplying by 9 (e.g., for $9 \times 6$, think $10 \times 6 = 60$, and $60 - 6 = 54$; for $7 \times 9$, think $7 \times 10 = 70$, and $70 - 7 = 63$)  
  - repeated doubling (e.g., if $2 \times 6$ is equal to 12, then $4 \times 6$ is equal to 24, and $8 \times 6$ is equal to 48)  
  - repeated halving (e.g., for $60 ÷ 4$, think $60 ÷ 2 = 30$ and $30 ÷ 2 = 15$)  
  - Recall the multiples of 0, 1, 2, 3, and 5 to 81 and related division facts.  
  - Recall the multiplication facts that are squares (1 × 1, 2 × 2, up to 9 × 9). |
| **Revisions** 5.N.3. Apply mental math strategies to determine multiplication and related division facts to 81 (9 × 9). [C, CN, ME, R, V] | - Describe the mental mathematics strategy used to determine a basic fact, such as  
  - skip-count up by one or two groups from a known fact (e.g., if $5 \times 7 = 35$, then $6 \times 7$ is equal to $35 + 7$ and $7 \times 7$ is equal to $35 + 7 + 7$)  
  - skip-count down by one or two groups from a known fact (e.g., if $8 \times 8 = 64$, then $7 \times 8$ is equal to $64 - 8$ and $6 \times 8$ is equal to $64 - 8 - 8$)  
  - halving/doubling (e.g., for $8 \times 3$ think $4 \times 3 = 12$, and $8 \times 3 = 12 + 12$)  
  - use patterns when multiplying by 9 (e.g., for $9 \times 6$, think $10 \times 6 = 60$, then $60 - 6 = 54$; for $7 \times 9$, think $7 \times 10 = 70$, then $70 - 7 = 63$)  
  - repeated doubling (e.g., if $2 \times 6$ is equal to 12, then $4 \times 6$ is equal to 24, and $8 \times 6$ is equal to 48)  
  - repeated halving (e.g., for $60 ÷ 4$, think $60 ÷ 2 = 30$ and $30 ÷ 2 = 15$)  
  - relating multiplication to division facts (e.g., for $7 \times 8$, think $56 ÷ 7 = 8$)  
  - use multiplication facts that are squares (1 × 1, 2 × 2, up to 9 × 9)  
  - Refine personal strategies to increase efficiency (e.g., 6 × 7 use known square $6 \times 6 + 6$ instead of repeated addition $6 + 6 + 6 + 6 + 6 + 6 + 6 + 6$). |

Recall of multiplication facts to 81 and related division facts is expected by the end of Grade 5.
## Skip Counting

<table>
<thead>
<tr>
<th>Current</th>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.N.1. Say the number sequence forward and backward from 0 to 1000 by</td>
<td>Extend a skip-counting sequence by 10s, or 100s, forward and backward, using a given starting point.</td>
<td>![C] Communication [CN] Connections [ME] Mental Mathematics and Estimation</td>
</tr>
<tr>
<td>■ 10s, or 100s, using any starting point</td>
<td>■ Extend a skip-counting sequence by 5s, forward and backward, starting at a given multiple of 5.</td>
<td></td>
</tr>
<tr>
<td>■ 5s using starting points that are multiples of 5</td>
<td>■ Extend a skip-counting sequence by 25s, forward and backward, starting at a given multiple of 25.</td>
<td></td>
</tr>
<tr>
<td>■ 25s using starting points that are multiples of 25</td>
<td>■ Identify and correct errors and omissions in a skip-counting sequence.</td>
<td></td>
</tr>
<tr>
<td>[C, CN, ME]</td>
<td>■ Determine the value of a set of coins (nickels, dimes, quarters, loonies) by using skip counting.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>■ Identify and explain the skip-counting pattern for a number sequence.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Revisions</th>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.N.1. Say the number sequence <strong>between any two given numbers</strong> forward and backward</td>
<td>Extend a skip-counting sequence by 10s, or 100s, forward and backward, using a given starting point.</td>
<td></td>
</tr>
<tr>
<td>■ from 0 to 1000 by</td>
<td>Extend a skip-counting sequence by 5s, forward and backward, starting at a given multiple of 5.</td>
<td></td>
</tr>
<tr>
<td>■ 10s or 100s, using any starting point</td>
<td>Extend a skip-counting sequence by 25s, forward and backward, starting at a given multiple of 25.</td>
<td></td>
</tr>
<tr>
<td>■ 5s, using starting points that are multiples of 5</td>
<td><strong>Extend a given skip-counting sequence by 3s, forward, starting at a given multiple of 3.</strong></td>
<td></td>
</tr>
<tr>
<td>■ 25s, using starting points that are multiples of 25</td>
<td><strong>Extend a given skip-counting sequence by 4s, starting at a given multiple of 4.</strong></td>
<td></td>
</tr>
<tr>
<td>■ from 0 to 100 by</td>
<td>Identify and correct errors and omissions in a skip-counting sequence.</td>
<td></td>
</tr>
<tr>
<td>■ 3s, using starting points that are multiples of 3</td>
<td>Determine the value of a set of coins (nickels, dimes, quarters, loonies) by using skip counting.</td>
<td></td>
</tr>
<tr>
<td>■ 4s, using starting points that are multiples of 4</td>
<td>Identify and explain the skip-counting pattern for a number sequence.</td>
<td></td>
</tr>
</tbody>
</table>

### Adding, Subtracting, Multiplying, and Dividing Whole Numbers

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td></td>
</tr>
<tr>
<td>4.N.3. Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by</td>
<td>Determine the sum of two numbers using a personal strategy (e.g., for 1326 + 548, record 1300 + 500 + 74).</td>
</tr>
<tr>
<td></td>
<td>using personal strategies for adding and subtracting</td>
</tr>
<tr>
<td></td>
<td>estimating sums and differences</td>
</tr>
<tr>
<td></td>
<td>solving problems involving addition and subtraction</td>
</tr>
<tr>
<td>[C, CN, ME, PS, R]</td>
<td></td>
</tr>
<tr>
<td>Revisions</td>
<td></td>
</tr>
<tr>
<td>4.N.3. Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals)</td>
<td>Model addition and subtraction using concrete materials and visual representations, and record the process symbolically.</td>
</tr>
<tr>
<td></td>
<td>concretely, pictorially, and symbolically by</td>
</tr>
<tr>
<td></td>
<td>using personal strategies</td>
</tr>
<tr>
<td></td>
<td>using the standard algorithms</td>
</tr>
<tr>
<td></td>
<td>estimating sums and differences</td>
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<td>solving problems</td>
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<tr>
<td>[C, CN, ME, PS, R]</td>
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Adding, Subtracting, Multiplying, and Dividing Whole Numbers (continued)

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<tr>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
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</thead>
<tbody>
<tr>
<td><strong>Current</strong> 5.N.5. Demonstrate an understanding of multiplication (2-digit numerals by 2-digit numerals) to solve problems. [C, CN, PS, V]</td>
<td>Illustrate partial products in expanded notation for both factors [e.g., for 36 × 42, determine the partial products for (30 + 6) × (40 + 2)]. Represent both 2-digit factors in expanded notation to illustrate the distributive property [e.g., to determine the partial products of 36 × 42, (30 + 6) × (40 + 2) = 30 × 40 + 30 × 2 + 6 × 40 + 6 × 2 = 1200 + 60 + 240 + 12 = 1512]. Model the steps for multiplying 2-digit factors using an array and base-10 blocks, and record the process symbolically. Describe a solution procedure for determining the product of two 2-digit factors using a pictorial representation, such as an area model. Solve a multiplication problem in context using personal strategies, and record the process.</td>
</tr>
<tr>
<td><strong>Revisions</strong> 5.N.5. Demonstrate an understanding of multiplication (1- and 2-digit multipliers and up to 4-digit multiplicands) concretely, pictorially, and symbolically by using personal strategies, using the standard algorithm, estimating products to solve problems. [C, CN, ME, PS, V]</td>
<td>Illustrate partial products in expanded notation for both factors [e.g., for 36 × 42, determine the partial products for (30 + 6) × (40 + 2)]. Represent both 2-digit factors in expanded notation to illustrate the distributive property [e.g., to determine the partial products of 36 × 42, (30 + 6) × (40 + 2) = 30 × 40 + 30 × 2 + 6 × 40 + 6 × 2 = 1200 + 60 + 240 + 12 = 1512]. Model the steps for multiplying 2-digit factors using an array and base-10 blocks, and record the process symbolically. Describe a solution procedure for determining the product of two 2-digit factors using a pictorial representation, such as an area model. Model and explain the relationship that exists between an algorithm, place value, and number properties. Determine products using the standard algorithm of vertical multiplication. (Numbers arranged vertically and multiplied using single digits which are added to form a final product.) Solve a multiplication problem in context using personal strategies, and record the process. Refine personal strategies such as mental math strategies to increase efficiency when appropriate (e.g., 16 × 25 think 4 x (4x 25) = 400).</td>
</tr>
</tbody>
</table>

[C] Communication [PS] Problem Solving
[CN] Connections [R] Reasoning
and Estimation [V] Visualization

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Adding, Subtracting, Multiplying, and Dividing Whole Numbers (continued)

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
</tr>
</thead>
</table>
| **Current** 5.N.6. Demonstrate an understanding of division (3-digit numerals by 1-digit numerals) with and without concrete materials, and interpret remainders to solve problems. [C, CN, PS] | ■ Model the division process as equal sharing using base-10 blocks, and record it symbolically.  
■ Explain that the interpretation of a remainder depends on the context:  
  ■ ignore the remainder (e.g., making teams of 4 from 22 people)  
  ■ round up the quotient (e.g., the number of five passenger cars required to transport 13 people)  
  ■ express remainders as fractions (e.g., five apples shared by two people)  
  ■ express remainders as decimals (e.g., measurement or money)  
■ Solve a division problem in context using personal strategies, and record the process. |
| **Revisions** 5.N.6. Demonstrate an understanding of division (1- and 2-digit divisors and up to 4-digit dividends) concretely, pictorially, and symbolically and interpret remainders by  
  ■ using personal strategies  
  ■ using the standard algorithm  
  ■ estimating quotients  
to solve problems. [C, CN, ME, PS] | ■ Model the division process as equal sharing using base-10 blocks, and record it symbolically.  
■ Explain that the interpretation of a remainder depends on the context:  
  ■ ignore the remainder (e.g., making teams of 4 from 22 people)  
  ■ round up the quotient (e.g., the number of five passenger cars required to transport 13 people)  
  ■ express remainders as fractions (e.g., five apples shared by two people)  
  ■ express remainders as decimals (e.g., measurement or money)  
■ Model and explain the relationship that exists between an algorithm, place value, and number properties.  
■ Determine quotients using the standard algorithm of long division. (The multiples of the divisor are subtracted from the dividend.)  
■ Solve a division problem in context using personal strategies, and record the process.  
■ Refine personal strategies such as mental math strategies to increase efficiency when appropriate (e.g., 860 ÷ 2 think 86 ÷ 2 = 43 then 860 ÷ 2 is 430). |
# Adding, Subtracting, Multiplying, and Dividing of Decimal Numbers

## Specific Learning Outcome(s)

### Current

5.N.11. Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).

[C, CN, PS, R, V]

### Revisions

5.N.11. Demonstrate an understanding of addition and subtraction of decimals (to thousandths) concretely, pictorially, and symbolically by:

- using personal strategies
- using the standard algorithms
- using estimation
- solving problems

[C, CN, ME, PS, R, V]

## Achievement Indicators

### Current

- Place the decimal point in a sum or difference using front-end estimation (e.g., for 6.3 + 0.25 + 306.158, think 6 + 306, so the sum is greater than 312).
- Correct errors of decimal point placements in sums and differences without using paper and pencil.
- Explain why keeping track of place value positions is important when adding and subtracting decimals.
- Predict sums and differences of decimals using estimation strategies.
- Solve a problem that involves addition and subtraction of decimals, limited to thousandths.

### Revisions

- Estimate a sum or difference using front-end estimation (e.g., for 6.3 + 0.25 + 306.158, think 6 + 306, so the sum is greater than 312) and place the decimal in the appropriate place.
- Correct errors of decimal point placements in sums and differences without using paper and pencil.
- Explain why keeping track of place value positions is important when adding and subtracting decimals.
- Predict sums and differences of decimals using estimation strategies.
- Solve a problem that involves addition and subtraction of decimals, to thousandths.
- Model and explain the relationship that exists between an algorithm, place value, and number properties.
- Determine the sum and difference using the standard algorithms of vertical addition and subtraction. (Numbers are arranged vertically with corresponding place value digits aligned.)
- Refine personal strategies, such as mental math, to increase efficiency when appropriate (e.g., 3.36 + 9.65 think 0.35 + 0.65 = 1.00, therefore, 0.36 + 0.65 = 1.01 and 3 + 9 = 12 for a total of 13.01).
### General and Specific Learning Outcomes

#### Adding, Subtracting, Multiplying, and Dividing of Decimal Numbers (continued)

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
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</table>
| **Current**                                                                                 | - Place the decimal point in a product using front-end estimation (e.g., for 15.205 m × 4, think 15 m × 4, so the product is greater than 60 m).  
  - Place the decimal point in a quotient using front-end estimation (e.g., for $26.83 ÷ 4, think $24 ÷ 4, so the quotient is greater than $6).  
  - Predict products and quotients of decimals using estimation strategies.  
  - Identify and correct errors of decimal point placement in a product or quotient by estimating.  
  - Solve a problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.  
  - Use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10 (e.g., 2.47 × 10 = 24.7; 31.9 ÷ 100 = 0.319). |
| 6.N.8. Demonstrate an understanding of multiplication and division of decimals involving      | ▪ 1-digit whole-number multipliers  
  ▪ 1-digit natural number divisors  
  ▪ multipliers and divisors that are multiples of 10  
  [C, CN, ME, PS, R, V]                                                                                                                                                                                                                                                                                     |
| **Revisions**                                                                               | - Estimate a product using front-end estimation (e.g., for 15.205 m × 4, think 15 m × 4, so the product is greater than 60 m), and place the decimal in the appropriate place.  
  - Estimate a quotient using front-end estimation (e.g., for $26.83 ÷ 4, think 24 ÷ 4, so the quotient is greater then $6), and place the decimal in the appropriate place.  
  - Predict products and quotients of decimals using estimation strategies.  
  - Identify and correct errors of decimal point placement in a product or quotient by estimating.  
  - Solve a problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.  
  - Use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10 (e.g., 2.47 × 10 = 24.7; 31.9 ÷ 100 = 0.319). |
| 6.N.8. Demonstrate an understanding of multiplication and division of decimals involving      | ▪ using personal strategies  
  ▪ using the standard algorithms  
  ▪ using estimation  
  ▪ solving problems  
  [C, CN, ME, PS, R, V]                                                                                                                                                                                                                                                                                     |
| (involving 1-digit whole-number multipliers, 1-digit natural number divisors, and multipliers and divisors that are multiples of 10) concretely, pictorially, and symbolically by |                                                                                                                                                                                                                                                                                                                                                       |
| ▪ using personal strategies  
  ▪ using the standard algorithms  
  ▪ using estimation  
  ▪ solving problems  
  [C, CN, ME, PS, R, V]                                                                                                                                                                                                                                                                                     |

### Skills and Knowledge

- **C**: Communication  
- **CN**: Connections  
- **ME**: Mental Mathematics  
- **PS**: Problem Solving  
- **R**: Reasoning  
- **T**: Technology and Estimation  
- **V**: Visualization
Adding, Subtracting, Multiplying, and Dividing of Decimal Numbers *(continued)*

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s)</th>
<th>Achievement Indicators</th>
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</table>
| **Current** 7.N.2. Demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected). [ME, PS, T] | ■ Solve a problem involving the addition of two or more decimal numbers.  
■ Solve a problem involving the subtraction of decimal numbers.  
■ Solve a problem involving the multiplication or division of decimal numbers (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected).  
■ Place the decimal in a sum or difference using front-end estimation (e.g., for $4.5 + 0.73 + 256.458$, think $4 + 256$, so the sum is greater than 260).  
■ Place the decimal in a product using front-end estimation (e.g., for $12.33 \times 2.4$, think $12 \times 2$, so the product is greater than $24$).  
■ Place the decimal in a quotient using front-end estimation (e.g., for $51.50 \div 2.1$, think $50 \div 2$, so the quotient is approximately $25$ m).  
■ Check the reasonableness of answers using estimation.  
■ Solve a problem that involves operations on decimals (limited to thousandths) taking into consideration the order of operations.  
■ Explain, using an example, how to use mental math for products or quotients when the multiplier or the divisor is 0.1 or 0.5 or 0.25. |
| **Revisions** 7.N.2. Demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, **technology could be used**). [ME, PS, T] | ■ Solve a problem involving the addition of two or more decimal numbers.  
■ Solve a problem involving the subtraction of decimal numbers.  
■ Solve a problem involving the multiplication or division of decimal numbers (for more than 1-digit divisors or 2-digit multipliers, **technology could be used**).  
■ Place the decimal in a sum or difference using front-end estimation (e.g., for $4.5 + 0.73 + 256.458$, think $4 + 256$, so the sum is greater than 260).  
■ Place the decimal in a product using front-end estimation (e.g., for $12.33 \times 2.4$, think $12 \times 2$, so the product is greater than $24$).  
■ Place the decimal in a quotient using front-end estimation (e.g., for $51.50 \div 2.1$, think $50 \div 2$, so the quotient is approximately $25$ m).  
■ Check the reasonableness of answers using estimation.  
■ Solve a problem that involves operations on decimals (limited to thousandths) taking into consideration the order of operations.  
■ Explain, using an example, how to use mental math for products or quotients when the multiplier or the divisor is 0.1 or 0.5 or 0.25. |


The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

<table>
<thead>
<tr>
<th>STRAND</th>
<th>GRADE</th>
<th>K</th>
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<td>Data Analysis</td>
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<td>Chance and Uncertainty</td>
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**NATURE OF MATHEMATICS**
- Change
- Constancy, Number Sense, Patterns, Relationships, Spatial Sense, Uncertainty

**MATHEMATICAL PROCESSES:**
- Communication, Connections, Mental Mathematics and Estimation, Problem Solving, Reasoning, Technology, Visualization

**GENERAL LEARNING OUTCOMES,** **SPECIFIC LEARNING OUTCOMES,** **AND ACHIEVEMENT INDICATORS**
Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip-counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Constancy is described by the terms stability, conservation, equilibrium, steady state, and symmetry. AAAS–Benchmarks 270)

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry. Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.
Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (BC Ministry of Education 146).

Number sense is an awareness and understanding of what numbers are, their relationships, their magnitude, and the relative effect of operating on numbers, including the use of mental mathematics and estimation (Fennell and Landis 187).

Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. Students who have number sense are computationally fluent, are flexible with numbers, and have intuition about numbers. Number sense evolves and typically results as a by-product of learning rather than through direct instruction. Number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics.

These skills contribute to students’ interaction with and understanding of their environment.

Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps develop students’ algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The discovery of possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.
Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.
Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The Common Curriculum Framework incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, and written and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated.

Through connections, students should begin to view mathematics as useful and relevant.
Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. “Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding. . . Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine 5).

**Mental Mathematics and Estimation [ME]**

Mental mathematics and estimation is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids. It improves computational fluency by developing efficiency, accuracy, and flexibility.

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein 442).

Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope V).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgments and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to know which strategy to use and how to use it.

To help students become efficient with computational fluency, students need to develop mental math skills and recall math facts automatically. Learning math facts is a developmental process where the focus of instruction is on thinking and building number relationships. Facts become automatic for students through repeated exposure and practice. When a student recalls facts, the answer should be produced without resorting to inefficient means, such as counting. When facts are automatic, students are no longer using strategies to retrieve them from memory.

**Problem Solving [PS]**

“Problem solving is an integral part of all mathematics learning” (NCTM, Problem Solving). Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you . . . ?” or “How could you . . . ?”, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.
In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior knowledge in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives, and develops confident, cognitive, mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes, and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Technology has the potential to enhance the teaching and learning of mathematics. It can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. **Students need to know when it is appropriate to use technology such as a calculator and when to apply their mental computation, reasoning, and estimation skills to predict and check answers.** The use of technology can enhance, although it should not replace, conceptual understanding, procedural thinking, and problem solving throughout Kindergarten to Grade 8. While technology can be used in Kindergarten to Grade 3 to enrich learning, it is expected that students will meet all outcomes without the use of calculators.

**Visualization [V]**

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world” (Armstrong 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and to know several estimation strategies (Shaw and Cliatt 150).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.
Strands

The learning outcomes in the Manitoba Curriculum Framework are organized into four strands across the grades, K–9. Some strands are further subdivided into substrands. There is one general learning outcome per substrand across the grades, K–9.

The strands and substrands, including the general learning outcome for each, follow.

Number
- Develop number sense.

Patterns and Relations

Patterns
- Use patterns to describe the world and solve problems.

Variables and Equations
- Represent algebraic expressions in multiple ways.

Shape and Space

Measurement
- Use direct and indirect measure to solve problems.

3-D Objects and 2-D Shapes
- Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Transformations
- Describe and analyze position and motion of objects and shapes.

Statistics and Probability

Data Analysis
- Collect, display, and analyze data to solve problems.

Chance and Uncertainty
- Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.
Learning Outcomes and Achievement Indicators

The *Manitoba Curriculum Framework* is stated in terms of general learning outcomes, specific learning outcomes, and achievement indicators.

General learning outcomes are overarching statements about what students are expected to learn in each strand/substrand. The general learning outcome for each strand/substrand is the same throughout the grades.

Specific learning outcomes are statements that identify the specific skills, understanding, and knowledge students are required to attain by the end of a given grade.

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success.

In this document, the word “including” indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase “such as” indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome.

Summary

The conceptual framework for K–8 mathematics describes the nature of mathematics, mathematical processes, and the mathematical concepts to be addressed in Kindergarten to Grade 8 mathematics. The components are not meant to stand alone. Learning activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes, and lead students to an understanding of the nature of mathematics through specific knowledge, skills, and attitudes among and between strands.
Instructional Focus

The *Manitoba Curriculum Framework* is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of learning outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Routinely incorporating conceptual understanding, procedural thinking, and problem solving within instructional design will enable students to master the mathematical skills and concepts of the curriculum.
- Integration of the mathematical processes within each strand is expected.
- Problem solving, conceptual understanding, reasoning, making connections, and procedural thinking are vital to increasing mathematical fluency, and must be integrated throughout the program.
- Concepts should be introduced using manipulatives and gradually developed from the concrete to the pictorial to the symbolic.
- Students in Manitoba bring a diversity of learning styles and cultural backgrounds to the classroom and they may be at varying developmental stages. Methods of instruction should be based on the learning styles and abilities of the students.
- Use educational resources by adapting to the context, experiences, and interests of students.

- Collaborate with teachers at other grade levels to ensure the continuity of learning of all students.
- Familiarize yourself with exemplary practices supported by pedagogical research in continuous professional learning.
- Provide students with several opportunities to communicate mathematical concepts and to discuss them in their own words.

“Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students” (Ontario 24).
Please view this document in colour.

The learning outcomes that have been revised are stated with the current learning outcome on the top of the chart followed by the proposed change directly underneath highlighted in yellow.

Brown type indicates an achievement indicator that has moved from one grade to another. Moved achievement indicators are now learning outcomes.